PHOKHARA version 9.3

November 2017

Abstract

Electron–positron annihilation into hadrons is one of the basic reactions of particle physics. The radiative return [1, 2] offers the unique possibility for a measurement of this quantity over a wide range of energies. The large luminosity of present ϕ- and B-factories easily compensates for the additional factor of α due to the emission of a hard photon. PHOKHARA is a Monte Carlo event generator which simulates this process at next-to-leading order (NLO) accuracy. Versions 1.0 and 2.0 were based on a NLO treatment of the corrections from initial-state radiation (ISR). Those are independent of the final-state channel. Version 3.0 incorporates NLO corrections to final-state radiation (FSR) for pion pair production. Version 4.0 of PHOKHARA includes nucleon pair production and NLO corrections to FSR for muon pairs. PHOKHARA version 5.0 included $\pi^+\pi^-\pi^0$ and kaon pair (K^+K^- and $K^0\bar{K}^0$) production as new channels, and radiative ϕ decay contributions to the reaction $e^+e^- \to \pi^+\pi^-\gamma$. Version 5.1 of PHOKHARA introduces minor improvements with respect to the version 5.0. In version 6.0 the reaction $e^+e^- \to \Lambda(\to \pi^-p)\bar{\Lambda}(\to \pi^+\bar{p})\gamma$ (in LO approximation) was added. The Λ- and $\bar{\Lambda}$-spin dependence and the correlation among their decay products is included. In version 7.0 new modelling of the 4-pion [3], 2-pion and 2-kaon hadronic [4] currents was implemented and J/ψ and $\psi(2S)$ contributions to 2-pion, 2-kaon, 2-muon final states were introduced [5, 4]. In version 8.0 the following features were added: the program generates also the mode with no photon emission, as measured in scan experiments. This mode contains ISR corrections at NNLO level [30]; the new hadronic channel ($\eta\pi^+\pi^-$) was added [30]. In version 9.0 complete NLO radiative corrections are available for the muon pair production mode [32]. In PHOKHARA9.1 a new model of nucleon form factors was implemented and the FSR corrections to the proton - antiproton final state were added both in the radiative return and scan modes [31]. In PHOKHARA9.2 the resonant production of the charmonium states χ_{c1} and χ_{c2} was added with the subsequent decays $\chi_{ci} \to J/\psi(\to \mu^+\mu-)\gamma$ [33].

In the present distribution (PHOKHARA9.3) the new channels in the scan (at NLO) and the radiative return (at LO) mode were added: $\mathbf{e^+e^- \to P\gamma, P = \pi^0, \eta, \eta'}$ [34].

1 Introduction

The first version of the Monte Carlo event generator (PHOKHARA version 1.0 [6]) incorporates ISR only at NLO [7], with $\pi^+\pi^-\gamma(\gamma)$ and $\mu^+\mu^-\gamma(\gamma)$ as final states, and was designed to simulate configurations with photons emitted at relatively large angles, $\theta^2 \gg m_e^2/s$. Its second version (PHOKHARA version 2.0 [8]) extends the validity of the program into the small angle region [9, 10], incorporates FSR at leading order (LO) for $\pi^+\pi^-$ and $\mu^+\mu^-$ final states, and includes four-pion final states (without FSR) in the formulation described in detail in [11].

The third version of the generator (PHOKHARA version 3.0 [12]), allows simultaneous emission of one photon from the initial state and one photon from the final state, requiring only one of them to be hard. This includes in particular the radiative return to $\pi^+\pi^-(\gamma)$ and thus the measurement of the (one-photon) inclusive $\pi^+\pi^-$ cross section. The influence of collinear lepton pair radiation has been investigated in [13, 14, 15].

Recent experimental results indeed demonstrate the power of the method and indicate that a precision of one per cent or better is within reach. In view of this progress a further improvement of our theoretical understanding seems to be required. To meet that requirements PHOKHARA version 4.0 [16, 17]), introduced production of nucleon pairs (proton-anti-proton and neutron-anti-neutron) as new channels, and FSR for muon pair production at NLO. Some improvements were also incorporated to better describe FSR at NLO for pion pair production. The effect of photon vacuum polarisation was also implemented.

PHOKHARA version 5.0 (and 5.1) was a continuation of that effort. New hadronic channels were introduced, mainly $\pi^+\pi^-\pi^0$ and kaon pairs (K^+K^- and $K^0\bar{K}^0$) [18], and in addition the radiative ϕ decay contributions to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ was incorporated [19] to improve description of that process when running at ϕ-factory (DAPHNE) energy. The old parametrisation of the pion form factor [20] was changed to one based on a dual resonance model [21]. This form factor was supposed to describe data up to $Q^2 \sim 10$ GeV^2. However as experimental results [22] published after [21] shown that it still underestimates the experimental values for large Q^2. The vacuum polarisation was taken from [23], however the contributions from narrow resonances ($\phi, J/\psi$ etc.), which should be parametrised separately, were taken out from the original code. Linear interpolation of the 'continuum' contribution is used instead in the region of narrow resonances, which were reintroduced as additional separate contributions (only ϕ for muon and pion pair production in the present version). Version PHOKHARA 5.1 introduces some improvements with respect to version 5.0 (details of the models of the radiative ϕ decays were changed). In version 6.0 the reaction $e^+e^- \to \Lambda(\to \pi^- p)\bar{\Lambda}(\to \pi^+\bar{p})\gamma$ (in LO approximation) was added. The Λ- and $\bar{\Lambda}$-spin dependence and the correlation among their decay products is included [25].

In the version 7.0 of PHOKHARA a new model for the 4-pion hadronic channel was introduced [3]. Also pion and kaon form factors were improved with the emphasis on a kinematical region of large hadron invariant masses [4]. Basing on the description of J/ψ and $\psi(2S)$ resonances from [5], their contributions to 2-pion, 2-kaon, 2-muon final states were introduced [4]. The accuracy of the method employed there is at a per-mile level for a typical detector used in measurements by the radiative return method with an energy resolution of about 10 MeV. This method is not adequate for the narrow resonance description with infinitely good detector resolution (purely theoretical case). For the details see [4]. A new version of the vacuum polarisation by Daisuke Nomura and Thomas Teubner (VP_HLMNT_v1_3nonr) [26, 27, 28] was implemented - the user can choose between the new and the old one. A comparison between the two options can be found in [29].

In the distribution PHOKHARA 8.0 the following features were added: the program generates also the mode with no photon emission, as measured in scan experiments. This mode contains ISR corrections at NNLO level [30]; the new hadronic channel ($\eta\pi^+\pi^-$) was added [30].

In the distribution PHOKHARA 9.0 complete NLO radiative corrections were implemented for the muon pair production mode [32]. The size of the radiative corrections missing in the previous versions does depend on the energy of experiments and on the event selection used. It is discussed in detail in [32].

In the distribution PHOKHARA9.1 a new model of nucleon form factors was implemented and the FSR corrections to the proton - antiproton final state were added both in the radiative return and scan modes [31]. The implementation of the FSR corrections to the proton - antiproton final state in version 9.1 supersedes the one released in version 8.0 and the versions 8.0 and 9.0 should not be used to calculate them.

In the distribution PHOKHARA9.2 direct, resonant production of χ_{c1} and χ_{c2} in electron- positron annihilation through two virtual photons was added with the subsequent decays $\chi_{ci} \to J/\psi(\to \mu^+\mu-)\gamma$ [33]. It leads to a measurable at BESIII resonant enhancement of the cross section if the model used is correct.

In the present distribution PHOKHARA9.3 the new channels were added: $e^+e^- \to P\gamma$, $P = \pi^0, \eta, \eta'$ [34]. This processes are simulated at the NLO in the case of the scan mode and at the LO in the case of the radiative return mode. Additionally we added a tool for testing the installation of the code. It is discussed in Section 5.

Many additional aspects of the radiative return and PHOKHARA Monte Carlo event generator are discussed in [35].

2 The program

PHOKHARA is written is FORTRAN 77. Real variables and functions are defined `real*8`. Complex numbers and functions are defined `complex*16`. The present distribution consist of the following files:

- **phokhara_9.3.f** : the main program. It allows using both unweighted and weighted events.
- **phokhara_9.3.inc** : defines some variables and `COMMON` blocks which are used by most of the subroutines in the main program.
- **input_9.3.dat** : all the constants and specific parameters needed for the generation are given through this file. The values of these input parameters can be varied by the user.
- **const_and_model_paramall9.3.dat** : all the constants and specific parameters needed for the modelling of the hadronic currents are given in this file. The values of these input parameters can be varied by the user in case he uses his own version(s) of the hadronic current(s).
- **vac_pol_hc1.f** : vacuum polarisation from Ref. [23] changed as described in the Introduction
- **common.f** : file containing `COMMON` blocks used by vac_pol_hc1.f
- **vp_hlmnt_v1_3nonr_hc.f** : vacuum polarisation from Ref. [26, 27, 28]; the demo part of the program was taken out.
- **seed.dat** : contains the seed used to initialise `RANLUX`, the double precision random number generator [24]. After each generation run a new seed is stored in this file.
- **ranlxd.c** : double precision `RANLUX` code (in C)[24]
- **ranlxd.h, ranlux_fort.c** : interface FORTRAN–C for RANLUX
- **testlxf.for** : test program for RANLUX
- **seed_prod.for** : seed 'production' for running RANLUX
- **guide.ps, notes.ps** : description of RANLUX
- **README** : short description how to compile and link PHOKHARA and test RANLUX
- **Makefile** : for compiling and linking PHOKHARA
- **sub-directory test2** : contains file **testscript**, which test the correctness of the "installation" of the code
- **sub-directory eemmg-lib** : contains libraries relevant for NLO corrections to $e^+e^- \to \mu^+\mu^-\gamma$.
- **IMPORTANT :** To prepare the program for running one has to prepare first libraries for a given system/computer. It is done in the directory eemmg-lib, where one has to run the Makefile (as is it is prepared for gfortran). Please read README in this directory for other compilers (like ifort, g77). After this, one has to use make in the PH9.3 directory.

Further information and updates of the program can be find in the following web address:
`http://ific.uv.es/~rodrigo/phokhara`

3 Input file

The interaction of the user with the program is made through the file **input_9.3.dat** and in case the user is altering models of the hadronic currents also through **const_and_model_paramall9.3.dat**.

It defines some physical constants (**const_and_model_paramall9.3.dat**) and the specific parameters needed for the generation (**input_9.3.dat**). The values of these parameters can be changed by the user.

`nges` – number of events which shall be generated. The number of events accepted and returned in the output file depends on the kinematical constraints, the energy of the collision and the generated final state. Typically, the acceptance rate varies between 30% and a few per cent.

`nm` – number of events used to scan the integrand and find its maximum. A preliminary scan is maid to find the maximum(s) of the integrand(s) before the true generation starts. The value of the maximum used in the generation phase is slightly greater than this approximated maximum.

`outfile` – name of the output file where the four-momenta of the particles of the accepted events are stored.

`iprint` – whether the four-momenta of the generated events is printed (`iprint=1`) or not (`iprint=0`) through subroutine `writeevent` in the output file `outfile`.

`ph0` – The choice between Born with no photon (scan mode) or one photon (radiative return mode).
(`ph0=-1`)– only Born with no photon emission is calculated;
(`ph0= 0`)– radiative return mode: Born corresponds to one photon emission;
(`ph0= 1`)– scan mode : Born corresponds to no photon emission

`nlo` – whether the program should provide predictions at:
for (`ph0= 0`): LO (`nlo=0`) or NLO (`nlo=1`);
for (`ph0= 1`): NLO (`nlo=0`) or NNLO (`nlo=1`).
For muon mode with (`nlo=1`) only two possibilities are allowed:
ISR only and complete NLO corrections.

`w` – energy cutoff of soft photon emission, normalised to the centre-of-mass energy. The physical result is independent of its value. Recommended value $w = 10^{-4}$. If one includes the narrow resonance(s) contributions the distributions depend on this cutoff, but this effect is not present, when the distributions are convoluted with a detector energy resolution of about 10 MeV or bigger. The integrated cross section is independent on the cutoff (see [4] for the details).

`pion` – which final state channel shall be simulated:

`pion=0` : $\mu^+\mu^-$
`pion=1` : $\pi^+\pi^-$
`pion=2` : $2\pi^0\pi^+\pi^-$
`pion=3` : $2\pi^+2\pi^-$
`pion=4` : $p\bar{p}$
`pion=5` : $n\bar{n}$
`pion=6` : K^+K^-
`pion=7` : $K^0\bar{K}^0$
`pion=8` : $\pi^+\pi^-\pi^0$
`pion=9` : $\Lambda(\to \pi^- p)\bar{\Lambda}(\to \pi^+\bar{p})$
`pion=10` : $\eta\pi^+\pi^-$
`pion=11` : $\chi_{c_1} \to J/\psi(\to \mu^+\mu^-)\gamma$; the QED background is also included [33]
`pion=12` : $\chi_{c_2} \to J/\psi(\to \mu^+\mu^-)\gamma$; the QED background is also included [33]
`pion=13` : $\pi^0\gamma$

pion=14 : $\eta\gamma$

pion=15 : $\eta'\gamma$

fsr – it should be set to zero for scan mode with the exception of the $p\bar{p}$ final state, where fsr=1 uses multiplicative Coulomb factor for FSR corrections. Other options are not allowed. For radiative return mode: only ISR is simulated (fsr=0), FSR is included at LO without ISR–FSR interference (fsr=1) or with ISR–FSR interference (fsr=2). The ISR–FSR interference is included only when the program is running in LO mode.

fsrnlo – includes (fsrnlo=1) or not (fsrnlo=0) simultaneous emission of one photon from the initial state and one photon from the final state, and the corresponding virtual corrections. Only in the two-pions, two charged kaons and two-muons modes.

pion=11 and pion=12 – these modes work only with the following setup of parameters ph0=0, nlo=0, fsr=0, ifsnlo =0 and narr_res =1

ivac – vacuum polarisation off (ivac=0) or on: (ivac=1 - [23]) or (ivac=2 - [26, 27, 28]) .

FF_Pion – choice between Breit-Wigner parametrisation of the pion form factor: old versions (as in PHOKHARA 6.0) Kühn-Santmaria parametrisation (FF_Pion=0) or Gounaris-Sakurai parametrisation (FF_Pion=1) and new version from [4] (FF_Pion=2).

f0_model – three different models for radiative ϕ-decays: KK model [19] (f0_model=0), no structure [19] (f0_model=1), radiative ϕ-decays not included (f0_model=2), Cesare Bini (KLOE) - private communication (f0_model=3).

FF_kaon – choice between kaon form factor: constrained model (0) [4], unconstrained model (1)[4] and as in PHOKHARA 6.0 (2)

narr_res: no narrow resonances (0)- it is not allowed for pion=11 and pion=12; J/ψ (1) or $\psi(2S)$ (2); (1) only for pion = 0,1,6,7,11,12; (2) only for pion = 0,1,6,7

FF_pp : choice between nucleon form factors: FF_pp=0 old [16] and FF_pp=1 new [31] form factors

FF_Pgg : choice between two versions of two-photon transition form factors for π^0, η, η': FF_Pgg=0 (fit 1) and FF_Pgg=1 (fit 2) from [34]

chi_sw: works only for pion=11 and pion=12; Radiative return(chi_sw=0), Chi production(chi_sw=1), Radiative return + Chi production (chi_sw=2)

be_r: works only for pion=11 and pion=12; (0) assumes exact CMS-Energy, (1) each beam energy (=CMS-Energy/2) is smeared with Gaussian distribution of the given variance $= beamres^2$

The remaining set of parameters defines the specific experimental settings:

E – centre-of-mass energy (GeV).

beamres – beam resolution (GeV).

q2min – minimal squared invariant mass of the system formed by the hadrons and the tagged photon (GeV^2).

q2_min_c – minimal squared invariant mass of the hadronic/muonic system (GeV^2).

q2_max_c – maximal squared invariant mass of the hadronic/muonic system (GeV^2).

gmin – minimal energy of the tagged photon (GeV).

phot1cut – lower cut on the azimuthal angle of the tagged photon (degrees).

phot2cut – upper cut on the azimuthal angle of the tagged photon (degrees).

pi1cut – lower cut on the azimuthal angle of the muons or hadrons (degrees).

`pi2cut` – upper cut on the azimuthal angle of the muons or hadrons (degrees).

All the kinematical cuts are given in the centre-of-mass system of the initial particles. The azimuthal angles are defined with respect to the positron momentum.

In modes pion=13, 14, 15: $Q^2_{min} = m^2_P$, $Q^2_{max} = s(1-2w)$ **and all angles in full range are used in the generation. All cuts should be put in the event selection procedures.**

The program offers the possibility of presenting various differential distributions as histograms. If this option is used, the name of the output file where the histograms are stored and the attributes of each histogram must be given. They are used for both weighted and unweighted event samples.

`title(i)` – title of histogram `i`

`xlow(i)` – lower edge in x for histogram `i`

`xup(i)` – upper edge in x for histogram `i`

`bins(i)` – number of bins for histogram `i`

where `i=1,...,20`.

The file `const_and_model_paramall9.3.dat` contains all constants and parameters (up to version 5.0 initialised in subroutine input or given in the input file discussed above), specific for models of the hadronic currents.

First set of parameters defines physical constants: coupling constants, masses, and decay widths. The following values are used by default:

$1/\alpha$ = 137.03599911 – fine structure constant

G_F = 1.1663787$\cdot 10^{-5}$ – Fermi constant

$\sin^2\theta_W$ = 0.23126 – θ_W is the Weinberg angle

m_e = 0.51099906 $\cdot 10^{-3}$ GeV – electron mass

m_p = 0.938271998 GeV – proton mass

m_n = 0.93956533 GeV – neutron mass

m_μ = 0.1056583568 GeV – muon mass

$m_{\pi^\pm}$ = 0.13957018 GeV – charged pion mass

m_{π^0} = 0.1349766 GeV – neutral pion mass

$m_{K^\pm}$ = 0.493677 GeV – charged kaon mass

m_{K^0} = 0.497614 GeV – neutral kaon mass

m_Λ = 1.115683 GeV – lambda mass

For the parametrisation of the form factors and narrow resonances contributions more constants are needed. Their values are explicitly assigned in the subroutine `input` and could be set by the user, if he wants to modify the default values, by changing the appropriate values in the file `const_and_model_paramall9.3.dat`.

4 Output

PHOKHARA presents the output information in several forms and saves it in different files.

From version PH9.3 the output is also written to the file phokhara.out.

The four-momenta of the particles of the accepted events are stored in the file given by `outfile`. The format of the output is determined by the subroutine `writeevent(pion)` and can be changed by the user. All the momenta are given in the centre-of-mass system of the colliding electron and positron.

The subroutine `inithisto` (`inithistoMC` for weighted events) books the histograms being based on the information given by the input file. The subroutine `endhisto` (`endhistoMC` for weighted events) fills the histograms at the end of the generation run and save the result in the output file. The histogram information is stored in the intermediate steeps of the calculation in the matrix `histo(i,j)` (`histoMC(i,j)` for weighted events), where `i` identifies the histogram number and `j` the bin and the user can use its favourite histograming tool simply modifying the subroutines `inithisto` and `endhisto`, where the initialisation and filling the histograms take place.

By default only the Q^2 distribution, where Q^2 is the squared invariant mass of the hadronic (muonic) system, is calculated. The contribution from single photon events is stored in histogram `1`. The contribution from two photon events is stored in histogram `2`. The final Q^2 distribution is given by the sum of both results. Other differential distributions can be defined though the subroutine `addiere(wgt,qq,i)` (`addiereMC(wgt,qq,i)` for weighted events), where `wgt` is the weight of the event, `qq` is the value of Q^2 and `i` is equal to 1 for single photon events and 2 for two photon events. The four-momenta of the events are given by the matrix `momenta(i,0:3)`, where `i=1,...,7`, for the positron, the electron, the two real photons (for single photon events `momenta(4,0:3)` is set to zero), the virtual photon converting into hadrons, the $\pi^+(\mu^+,\bar{p},\bar{n},K^+,K^0)$ and $\pi^-(\mu^-,p,n,K^-,\bar{K}^0)$ respectively. In the three pion channel `i=6,7,8` for the $\pi^+\pi^-\pi^0$ respectively. For $\eta\pi^+\pi^-$ channel `i=6,7,8` for the $\eta\pi^+\pi^-$ respectively and in the four pion channels `i=6,...,9` for the $\pi^0\pi^0\pi^-\pi^+$ and $\pi^+\pi^-\pi^-\pi^+$ respectively. In the $\Lambda(\to\pi^-p)\bar{\Lambda}(\to\pi^+\bar{p})$ channel `i=8,...,11` for the $\pi^+\bar{p}\pi^-p$ respectively. For technical reasons the `i=6,7` correspond to $\bar{\Lambda}\Lambda$ respectively. In the $\chi_{c_{1,2}}\to J/\psi(\to\mu^+\mu^-)\gamma$ channels `i=6,7` correspond to $\mu^+\mu^-$ respectively and `i=3` to the photon. In the $e^+e^-\to P\gamma$ channels the `i=6` corresponds to P and `i=3,7` correspond to the photons momenta. This matrix can be used by the user to define other differential distributions.

At the end of the run, PHOKHARA displays also the total number of accepted events, the value of the cross section, the value of the scanned maximum(s) and the biggest value of the integrand(s) found during the Monte Carlo generation. The last should be always smaller than the scanned maximum. If during the generation a value of the integrand is found to be bigger than the scanned maximum, a warning is given. Then, the number of events used for the initial scan should be increased.

For the muon pair final state, when running with complete NLO mode [32] there might be problems with negative weights in generation for high energies. In this case one cannot use the unweighted event sample.

5 Testing the installation

Execution of the testing program can be done by running the script **testscript** in directory /test2 . It tests the correctness of the 'installation' of the code. The user have to supply the version of Phokhara to be tested. The script compares the previously generated "pure" output of the program for different channels/parameters, with those, which are produced by the user generated EXE file phokhara. If all of the tests have passed the following message is printed:
"All tests passed. OK."
In the case some of the tests failed the following message is printed:
"Some of tests failed
There were 'no. of errors' errors.
Please see the above messages for the details."
If for some of the channels the outputs are different the following message is printed: "Result: failed. Histograms are different." and the output files are stored in the fail.mode(name) directory for a manual inspection. In case the program does not run for some channel, the following message is printed: "Result: failed. PHOKHARA exited abnormally."
In case something goes wrong with the tests please contact the authors.

6 Forthcoming features

- Full one-loop radiative corrections for pion pair production.
- Simulation of other exclusive hadronic channels.

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